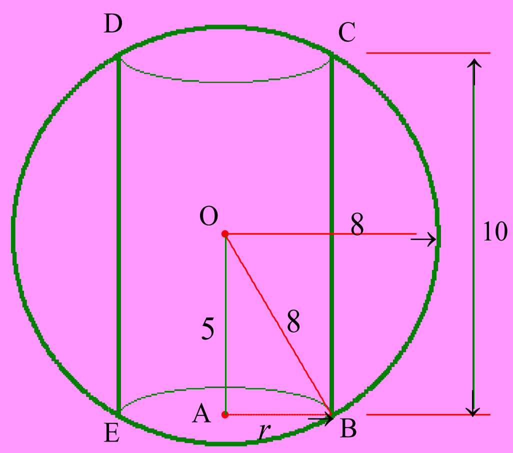
Concept based and Exam oriented

TUTORIAL LESSONS

Mathematics Year 9



In This Book

Fully worked solutions to all questions

Written explanation of theory and concepts for each question

Over 1000 specifically designed study and exam style questions

Strong focus on algebra from basic to the highest complexity

Answers to most of your questions

Help for homework and assessment

Exam preparation through each lesson

Systematic revision while study new topics

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About the book

Even though this book is fully compatible with the Victorian school Curriculum and contains all important topics and can also be very useful as supplementary study material in other states, the book is still different than any school text book as well as from most of the other mathematics books because all questions in this book are fully worked out and concisely explained in full details.

This book is designed to help you to find answers to, almost, all of your questions. Written by tutor with over 30 years tutoring experience, the book contains over 1000 fully worked questions with concise theoretical and conceptual explanation for each question. The intellectual base for this book is a systematic collection of questions asked by students over a long period of time. In order to provide students with all subject requirements the author created questions by adding other mathematical aspects such as: plenty algebra, application of mathematical theory and development of mathematical concepts and all this with strong tests and exam orientation. Finally, the wide range of different style of questions is helping students to learn mathematics with deeper understanding of theory and concepts that can be applied across the whole range of questions, rather than trying to apply some memorized patterns that usually can work on some certain questions. So this book will, definitely, help you:

- To find answers to most of your questions (even the most complex once)
- To do your homework and school tasks with easy and satisfactory way
- To be ready for your school assessments and final year exam
- To improve algebra skills
- To develop independent studying skills
- To get knowledge of the highest standard in the fastest time
- To understand and memorise even the most complex concepts in an easy way because most of the concepts are broken into simple steps and constantly revised throughout the book.
- To develop a high ability to apply theoretical knowledge and concepts in solving questions.
- To extend your educational ability and to increase motivation as you will discover that “knowledge is the power” and studying is not such hard work.

The book contains all important topics for Year 9 Mathematics, such as:

- Arithmetic and Algebra
- Linear Equations and Graphs
- Quadratic Expressions, Equations and Functions
- Index Laws and Surds
- Pythagoras Theorem and Trigonometry

There are 41 tutorial lessons. Each lesson (session), contains in average about 25 fully worked out questions with concise explanation of theory and concept applied for each question. The firs (or first few) sessions of each topic were designed to present theory and basic concepts for that topic. The other sessions are practices sessions giving to students the best examples to get full understanding of the subject. Questions are not arranged according to their level of complexity, because all questions are fully worked out and concisely explained so that every student (regardless of current level of knowledge) can easily understand algebra, concept and theory applied on each questions. The other reason why questions are not graded is to motivate students to work across the entire book rather than to work selectively. Such arrangement and very detailed explanation of questions enable to students of all levels of knowledge to progress with this book to the highest possible standard.

Finally, this book is an attempt to save student's time and money seeing tutors around and to make available to everyone the best tutorial lessons that has been delivered to students over a few decades.

A short author's message about how to study Mathematics

Choosing the right way of doing something is very solid base for success. As studying is a complex activity, so we have to be more conscious of choosing the right way of studying. This way is, generally, different from subject to subject and as Mathematics is one of very specific and quite complex subject, we have to take the right approach to that subject to ensure our success.

Definitely, we can't study Mathematics by reading even not by memorising simple facts and formulas. Studying Mathematics requires permanent practice over long period. As we are progressing from grade to grade, we have to keep fresh our previous knowledge (theory, concepts, even facts and formulas) and to build up that knowledge from current program.

When we start studying a new chapter, of course, we have to "read" the book. During "reading" time we have to stop "reading" whenever is necessary to write down important words, facts and formulas or to draw a graph or some other appropriate drawing, even if that graph or drawing is already in the book. During that time we have to identify the meaning of new terminology, maybe some formulas and definitions, key facts, theoretical explanations, probably some new concepts and finally to identify previously learned knowledge we should apply in the current chapter. When you are happy with your understanding of what you "read" then you can move to the next phase of studying. That is practicing. But how someone can be sure that his/her understanding is good enough to move to practicing? Here I wouldn't suggest any quiz or simple test questions trial. Instead, I would recommend a different technique. That is to visualise in front of you your friend, cousin or your parents and try to teach them of what you have been learning during "reading" time. If necessary, go back in book to refresh your memory or to check if you have missed something. The theory of each chapter of most of the books is usually written on just a few pages, so that "reading" process shouldn't consume long time. One or two hours should qualify you to move to practicing phase.

Once when you start practicing, you can start from easier questions and go to the most complex once. For each question, before you start working, spend few minutes time to think about an appropriate concept and draw a simple strategy how to apply that concept. When you start work make sure that your work is inside your concept and strategy drawn at the beginning. If the concept, you have, doesn't work for that question you should try to find out the clear reason why that concept is not good. That will help you to correct the concept and satisfactory to finish the question.

When due to move on another chapter, you should select some testing questions and to conduct short test of your understanding of current chapter. It is very useful, at the end of studying the chapter, once again, pretend to be a teacher and try to teach somebody. Compare your way of "teaching" somebody after "reading" phase and after practising phase of the end of studying the chapter and you will discover how big progress you made.

How to use this book

Thinking about method of how to study Mathematics, you already may have some ideas how to use this book. Even though you can use this book independently from any other books, still it would be better to study the chapter in your school and when you already have some knowledge of the chapter then to start using this book. Go over first session of the chapter to establish solid understanding of theory and important concepts. Even though you have all questions fully worked out, I am suggesting you to try to solve questions before checking working solution. After each question even if you solve the question correctly, go through worked solution with high attention as always there will be something there that you may learn. It is, also, recommended when going through worked solutions, to frequently reefer to theoretically session(s) to learn how the author is applying theory and concepts on practical questions.

Let start

Good luck

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SESSION 8

ALGEBRA

Practice 4

Session 8 - Algebra – Practice 4

1. $\frac{x+3}{4}-\frac{x-1}{4}$

2. $\frac{2x+1}{4a}+\frac{3x+1}{4a}-\frac{x-2}{4a}$

3) $\frac{1-a}{x-y}-\frac{1-3a}{x-y}$

4. $\frac{x-3}{x-5}+\frac{x+3}{5-x}$

5. $\frac{p}{a^2-1}-\frac{q}{1-a^2}$

6. $\frac{5a-3}{3}+\frac{3a-2}{5}$

7. $\frac{x}{2a+2b}-\frac{6x}{4a+4b}$

8. $\frac{1}{6a}+\frac{1}{4b^2}-\frac{a^2+b^2}{6ab^2}$

9. $x+y-\frac{x^2-4y^2}{x+2y}$

10. $\frac{4-2x+x^2}{2+x}-x-2$

11. $\frac{a}{ab-b^2}+\frac{b}{a^2-ab}-\frac{a+b}{ab}$

12. $\frac{a}{a^2-9b^2}-\frac{1}{a+3b}$

13. $\frac{16x-x^2}{x^2-4}+\frac{3+2x}{2-x}-\frac{2-3x}{x+2}$

14. $\frac{1}{x^2-x}+\frac{2}{1-x^2}-\frac{1}{x^2+x}$

15. $\frac{1}{6x+3}-\frac{9x+3}{8x^2-2}+\frac{2}{2x-1}$

16. $\frac{5x}{x^2-6x+9}-\frac{3x-1}{x^2-9}$

17. $\frac{5}{2y^2+6y}-\frac{4-3y^2}{y^2-9}-3$

18. $\frac{x^2+y^2}{xy}-\frac{x^2}{xy-y^2}+\frac{y^2}{x^2-xy}$

19. $\left(\frac{1}{m}-\frac{1}{n}\right)\div\left(\frac{1}{m^2}-\frac{1}{n^2}\right)$

20. $\left(1-\frac{3x^2}{1-x^2}\right)\div\left(\frac{x}{x-1}+1\right)$

21. $\frac{y-\frac{a^2}{y}}{a-\frac{y^2}{a}}$

22. $\frac{4-\frac{4+a^2}{a}}{\frac{1}{2}-\frac{1}{a}}$

23. $\frac{1-\frac{2b}{a}+\frac{b^2}{a^2}}{a-b}$

24. $2+\frac{\frac{1}{x-1}-\frac{1}{x+1}}{x+\frac{x}{x^2-1}}$

25. $\frac{2x+1-\frac{1}{2x+1}}{1+\frac{1}{2x+1}}$

26. $\frac{1+a+\frac{1}{1-a}}{1+\frac{1}{1-a^2}}$

Session 8 - Solutions to Algebra Practice 4

1. Take care about negative sign of fractional term $\left(-\frac{x-1}{4}\right)$ especially if numerator has two or more terms. ***You must change signs of all terms in numerator.***

$$\frac{x+3}{4}-\frac{x-1}{4}=\frac{x+3-x+1}{4}=\frac{4}{4}=1$$

2. $$\frac{2x+1}{4a}+\frac{3x+1}{4a}-\frac{x-2}{4a}=\frac{2x+1+3x+1-x+2}{4a}=\frac{4x+6}{4a}=\frac{2(2x+3)}{4a}=\frac{2x+3}{2a}$$

3. $$\frac{1-a}{x-y}-\frac{1-3a}{x-y}=\frac{1-a-1+3a}{x-y}=\frac{2a}{x-y}$$

4. If two denominators are opposite in sign, it is recommended to take – 1 as a common factor from one of denominators and to make them the same. Don't forget to change the sign of fractional term in which you are changing denominator.

$$\frac{x-3}{x-5}+\frac{x+3}{5-x}=\frac{x-3}{x-5}+\frac{x+3}{-(x-5)}=\frac{x-3}{x-5}-\frac{x+3}{x-5}=\frac{x-3-x-3}{x-5}=-\frac{6}{x-5}$$

5. $$\frac{p}{a^2-1}-\frac{q}{1-a^2}=\frac{p}{a^2-1}-\frac{q}{-(a^2-1)}=\frac{p}{a^2-1}+\frac{q}{a^2-1}=\frac{p+q}{a^2-1}$$

6. $$\frac{5a-3}{3}+\frac{3a-2}{5}=\frac{5(5a-3)+3(3a-2)}{15}=\frac{25a-15+9a-6}{15}=\frac{34a-21}{15}$$

7. Simplify expressions in each step of work (wherever is possible). Remember, any factorized form is simpler than expanding form. Always look for common factors and if there is anything to cancel and if there is possibility for that, do it.

$$\frac{x}{2a+2b}-\frac{6x}{4a+4b}=\frac{x}{2(a+b)}-\frac{6x}{4(a+b)}=\frac{x}{2(a+b)}-\frac{3x}{2(a+b)}=\frac{x-3x}{2(a+b)}=$$

$$=-\frac{2x}{2(a+b)}=-\frac{x}{a+b}$$

SESSION 9

Pythagoras Theorem

Definition & Practice Examples

Session 9 – Pythagoras theorem practice examples

1.

2.

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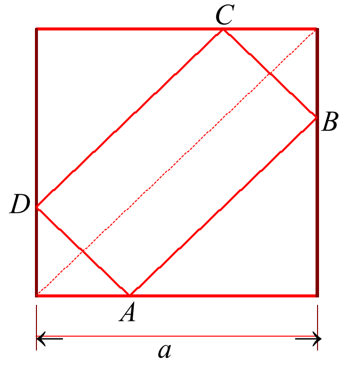
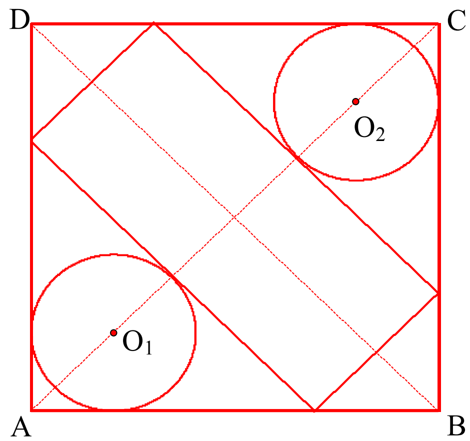
4.

5. The rectangle whose perimeter is $20\sqrt{2}$ cm is inscribed into a square of side a cm. Find the length of side a of this square.

   Note: Rectangle is inscribed into square if each of 4 vertexes are on each side of a square. Two parallel sides of such rectangle are parallel with diagonal of a square.



6. Two circles of the same diameter are placed inside the square so that their centres are on a diagonal of the square. The rectangle is also placed inside the square as shown on the diagram. Find the area of the rectangle if diameter of circles is 12 cm and diagonal of the square is 42 cm.



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12.

Session 9 – Solutions to Pythagoras theorem examples

1.

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5. We have to start from formula for perimeter of rectangle (the only known) and later to apply Pythagoras theorem to find length (l) and width (w) of rectangle in terms of a.

$P = 2l + 2w$

$20\sqrt{2} = 2l + 2w \quad / \div 2$

$10\sqrt{2} = l + w$

From triangle ΔDCH

$l^2 = (a-x)^2 + (a-x)^2$

$l^2 = 2(a-x)^2$

$l = \sqrt{2(a-x)^2}$

$l = (a-x)\sqrt{2}$

From triangle ΔBGC

$w^2 = x^2 + x^2$

$w^2 = 2x^2$

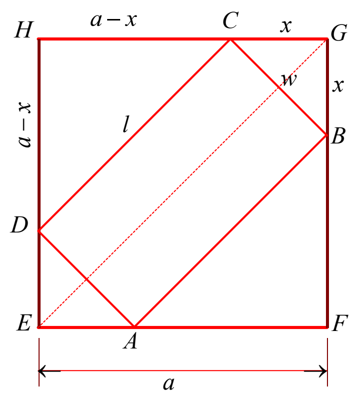
$w = \sqrt{2x^2}$

$w = x\sqrt{2}$

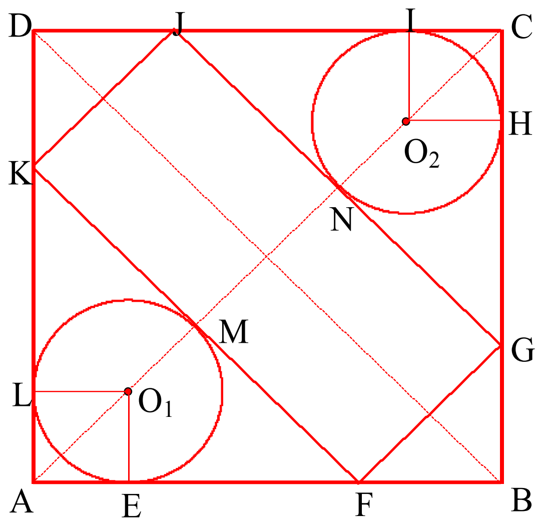
$10\sqrt{2} = (a-x)\sqrt{2} + x\sqrt{2} \quad / \div \sqrt{2}$

$10 = a - x + x$

$a = 10 \text{ cm}$



6. This question looks a bit complex, but if we follow common concept for questions involving Pythagoras theorem then we can solve this question much easier than it looks. So the concept is to identify right angled triangles for which we know 2 sides and by applying Pythagoras theorem to calculate third sides. To avoid some unnecessary calculation it is important to identify only right angled triangles that we need. The easiest way to see those triangles is to start with formula for area of this rectangle as the area of that rectangle is required by the question. Starting with direct answer to the question doesn't help us only to see triangles that we need to calculate, but also it will give us right guidance throughout entire work.



To make our work even more easily, it is good idea to mark all critical points on our diagram (we shouldn't think only about points that we will need. Simply mark all of them). We also can see from diagram that some lengths are the same and we can assign a letter for each length we need to use. So, let

$\overline{KF} = \overline{JG} = l$	Length of rectangle
$\overline{FG} = \overline{KJ} = \overline{MN} = w$	Width of rectangle
$\overline{AB} = \overline{BC} = a$	Side of square
$\overline{LO_1} = \overline{EO_1} = \overline{MO_1} = \overline{HO_2} = \overline{IO_2} = \overline{NO_2} = r = 6$	Radius of the circle (diameter is given)
$\overline{AC} = 42$	Diagonal of a square
$\overline{FB} = \overline{BG} = \overline{DJ} = \overline{DK} = x$	
$\overline{AF} = \overline{AK} = \overline{CG} = \overline{CJ} = y$	
$\overline{AO_1} = \overline{CO_2} = b$	

Now when we have all necessary analysis we can start working out this question. We said, we will start from formula for area of the given rectangle. So

7.

8.

9.

10.

11.

12.

S E S S I O N 26

LINEAR GRAPH

Concepts and Examples

Straight Line Graph

Terminology and definitions

Mathematical expression of the form $ax + b = c$ where *a, b* and *c* are any numbers and *x* is variable, we are calling ***equation***. In this case linear equation because the highest power of *x* is 1.Another, similar, expression $y = mx + c$ is also a linear equation. If *x* and *y*, in this equation, are variables while *m* and *c* are any numbers, then this equation represents a rule that telling us how *y* changes when *x* changes. Such equation is, so called, ***linear relation or function***. In this relation *x* is ***independent variable*** and *y* is ***dependent variable***.

Consider the following statement. *The total cost of hiring a car is $15 fixed administrative cost, plus $0.5 for each kilometre of driving.*

This statement is, actually, a rule for the total cost of hiring a car and we can calculate the cost for our trip. This statement (rule) we can translate in mathematical language using numbers and symbols. If we donate *C* for the total cost in dollars and *x* for kilometres driving, then this rule we can write as $C = 0.5x + 15$. We can see that this rule has the same form as our expression $(y = mx + c)$ from the top of this page that we called ***linear relation or function.***

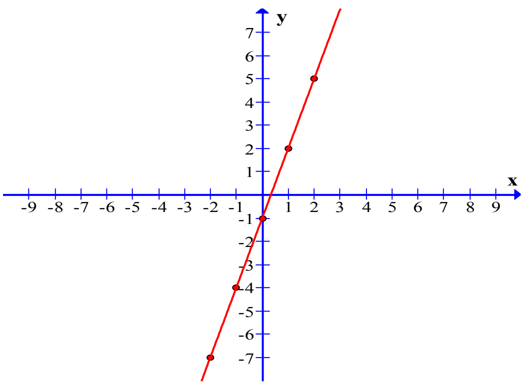
Any relation can be expressed by equation that connects *x* and *y* or by ***graph***. Since graph and equation describe the same rule, then we should understand how to sketch the graph when we know equation, as well how to write equation when we know the graph. In this topic we are talking about linear graph, and that is straight line that can be defined as a shortest distance between two points. The line is formed by infinite number of points that satisfied the same rule and each position of each point defined by ordered pairs of *x* and *y* coordinates in Cartesian or number plane.

Example 1

For linear function $y = 3x - 1$, for example, we can chose some values of *x* and calculate corresponding values of *y* to get ordered pairs of coordinates and then to plot each point. All points will lie on a straight line and that line is graph of $y = 3x - 1$. So

$$x = -2 \rightarrow y = 3x - 1 \Rightarrow y = 3 \times (-2) - 1 = -7$$
$$x = -1 \rightarrow y = 3x - 1 \Rightarrow y = 3 \times (-1) - 1 = -4$$
$$x = 0 \rightarrow y = 3x - 1 \Rightarrow y = 3 \times 0 - 1 = -1$$
$$x = 1 \rightarrow y = 3x - 1 \Rightarrow y = 3 \times 1 - 1 = 2$$
$$x = 2 \rightarrow y = 3x - 1 \Rightarrow y = 3 \times 2 - 1 = 5$$

x	−2	−1	0	1	2
y	−7	−4	−1	2	5



Similar, we can write equation of straight line if we know graph of that line. Of course, we have to know coordinates of at least 2 points on that line. Let see how this works on example below.

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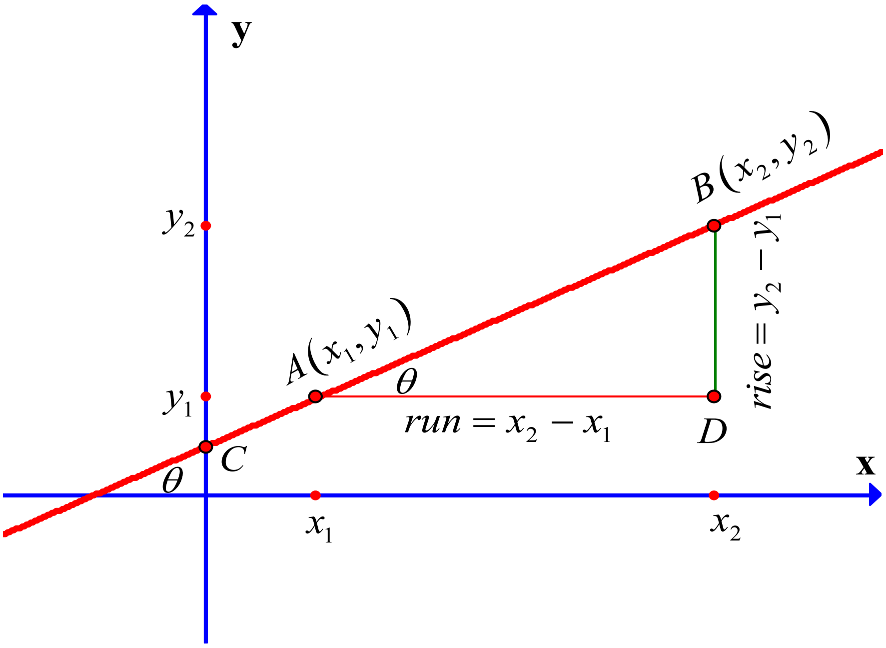
We saw at the beginning that gradient is difference between two consecutive y coordinates, but generally gradient is a ratio between vertical and horizontal distance between any two points on the line. Usually horizontal distance we are calling ***run*** while vertical distance ***rise***.

$$Gradient = \frac{\text{Vertical distance between any 2 points on the line}}{\text{Horisontal distance betwee those 2 points}} = \frac{rise}{run}$$

Generally, the equation of straight line in its standard form is given by $y = mx + c$. In this relation m is ***gradient*** (number that multiplies x) and c is ***y intercept.*** We can express this relation graphically as shown below



We can see from the graph that gradient is $m = \frac{rise}{ruun} = \frac{y_2 - y_1}{x_2 - x_1}$

Also triangle ABD is right angle triangle and from that triangle $\tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$. Since

$\frac{y_2 - y_1}{x_2 - x_1} = m$, than $\tan\theta = m$ where θ is angle that line makes with positive direction of x axis and m gradient of a line. The last formula ($\tan\theta = m$) also can be used to find the angle when we know equation of a line by $\theta = \tan^{-1} m$

x intercept is the point on the line where line cuts the x axis and y coordinate of that point is 0. **y intercept** is the point where line cuts the y axis and x coordinate of that point is 0.

Sketching and analysing graphs using table of ordered pairs of coordinates is time consuming and sometimes can be very hard. On next few pages we will learn simpler but more scientific way of sketching straight line graph and writing equation of straight line. That approach will be used not only in this course, but also throughout coming years of mathematics.

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S E S S I O N 37

Q U A D R A T I C S

Practice 3

Session 37 - Quadratics – Practice 3

1. Solve the following quadratic equations (use null factor law)

a) $-9=-x^2$ b) $\frac{4x^2}{3}=\frac{25}{3}$

c) $x^2-13x+42=0$ d) $x^2+16x+64=0$

e) $x^2+8x+12=0$ f) $2x^2+9x+4=0$

g) $-4x^2+12x-8=0$ h) $4x^2=10x-4$

2. Solve the following equations applying null factor law and check your answer using quadratic formula

a) $(x-1)^2+4(x-1)+3=0$ b) $(x-a)(2b-3x)=0$

c) $3ax^2-3bx=ax-b$ d) $x^2\sqrt{3}-x-2\sqrt{3}=0$

3) Determine the number and type of solutions of the following equations (do not solve them)

a) $x^2+6=12x$ b) $\frac{x^2+1}{2}-\frac{3-x}{4}=\frac{1}{3}$

c) $x^2+x+4=0$ d) $\frac{1}{3}(x^2-3)=x-1$

e) $\left(x-\frac{1}{2}\right)^2=2+3\left(x+\frac{5}{3}\right)$ f) $x^2+1=0$

4. Determine the value(s) of k for which the equation $x^2+8x+k=0$ will have:

a) one solution b) two solutions c) no solutions

5. For the line $y = 2x + c$ and parabola $y = x^2 - 4x + 1$, find value(s) of c so that:

a) line is a tangent to this parabola (touches parabola just at 1 point). Hence, find the coordinates of this point and sketch both lines on the same set of axes.
b) line intersect parabola (there are 2 common points)
c) line and parabola don't have any common point.

6. Find the coordinates of turning point, x and y intercepts and sketch the graph of the following parabolas.

a) $y = -2(x-1)^2 + 4$ b) $y = x^2 - 4x + 6$

c) $y = 2x^2 - 3x + 1$ d) $y = \frac{x^2}{2} + \frac{x}{2} - \frac{7}{8}$

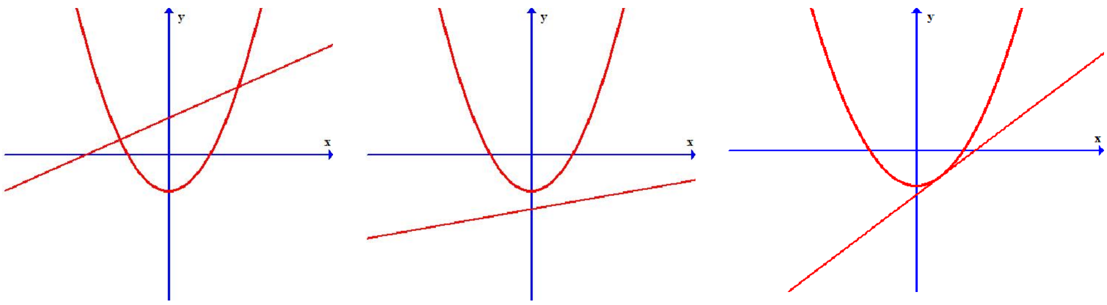
Session 37 - Quadratics – Solutions to Practice 3

1.
2.
3.
4.

- 321 -

5. This question requires understanding of relation between straight line and parabola. Basically, we can have 3 different situation of that relation:
   - Straight line may cut parabola at two points
   - Straight line may just touch parabola at one point. If straight line just touches parabola at one point then that straight line we are calling ***tangent*** to parabola.
   - Straight line doesn't have any common points with parabola.

If D > 0 | If D < 0 | If D = 0



To determine this relation we need to use discriminante, but before that we need to form quadratic equation for which we should calculate discriminante. Since any point of contact between two lines is a property of both lines, then we have to equalise equation of line with equation of parabola to form a quadratic equation on which we should apply discriminante. So we are starting this type of questions on the same way like we are looking for intersection points between two lines, but we are not calculating coordinates of those points, we are calculating discriminante .

a) $y = 2x + c$ $\qquad$ $y = x^2 - 4x + 1$

$x^2 - 4x + 1 = 2x + c$

$x^2 - 4x + 1 - 2x - c = 0$

$x^2 - 6x + 1 - c = 0$

We are not solving this equation for x (that would be if we are looking for coordinates of intersection points). We need to form discriminante for this equation and to equalise discriminante with zero because this equation should have only one solution as the line and parabola have only one point of contact (line is a tangent). Equation formed from discriminante is equation in c and from that equation we will calculate value of c.

$D = 0$

$(-6)^2 - 4(1 - c) = 0$

$36 - 4 + 4c = 0$

$4c = -32$ $\quad$ and $\quad$ $c = -8$

6.

S E S S I O N 41

ALGEBRA INVOLVING QUADRATICS

Practice 2

Session 41 – Algebra involving quadratics

Practice worksheet 2

Simplify the following

1. $\frac{3}{x^2+7x+10}+\frac{4}{x+2}$

2. $\frac{2}{x+4}+\frac{x-9}{16-x^2}-\frac{x-3}{x^2-8x+16}$

3. $\frac{(ax+1)^2-(x+a)^2}{(1-x^2)(1-a^2)}$

4. $x+y-\frac{x^2-y^2}{x+2y}$

5. $\frac{1-\frac{a}{2}}{a-\frac{1}{2}}$

6. $\left(\frac{a+b}{a-b}+\frac{a-b}{a+b}\right)^2-\left(\frac{a+b}{a-b}-\frac{a-b}{a+b}\right)^2$

7. Rationalise the denominator of $\frac{\sqrt{a}-\sqrt{2}}{\sqrt{a}+\sqrt{2}}$.

8. Show that $\left[\frac{n(n+1)}{2}\right]^2-\left[\frac{n(n-1)}{2}\right]^2=n^3$

9. Simplify $\frac{x-x^2}{1-x^2}+\frac{1+x}{1+2x+x^2}-\frac{1-2x}{1-x}$. Hence, find value for this expression if

   a) $x=0$ b) $x=1$ c) $x=\frac{1}{1-\sqrt{3}}$

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Session 41 – Algebra involving quadratics

Solutions to practice worksheet 2

1. $$\frac{3}{x^2+7x+10}+\frac{4}{x+2}$$

$$x^2+7x+10=x^2+2x+5x+10=x(x+2)+5(x+2)=(x+2)(x+5)$$

$$\frac{2}{(x+2)(x+5)}+\frac{4}{x+2}=\frac{2+4(x+5)}{(x+2)(x+5)}=\frac{2+4x+20}{(x+2)(x+5)}=\frac{2(2x+11)}{(x+2)(x+5)}$$

2. $$\frac{2}{x+4}+\frac{x-9}{16-x^2}-\frac{x-3}{x^2-8x+16}$$

Note: It is better if we take -1 as a common factor from denominator in second fraction and to have $-\frac{x-9}{x^2-16}$ because denominator in third fraction is a perfect square $(x-4)^2$ and new denominator in second fraction will give as the same factor $x-4$. So we have:

$$\frac{2}{x+4}-\frac{x-9}{x^2-16}-\frac{x-3}{x^2-8x+16}=\frac{2}{x+4}-\frac{x-9}{(x-4)(x+4)}-\frac{x-3}{(x-4)^2}=$$

$$=\frac{2(x-4)^2-(x-9)(x-4)-(x-3)(x+4)}{(x+4)(x-4)^2}=$$

$$=\frac{\cancel{2x^2}-16x+32\cancel{-x^2}+13x-36\cancel{-x^2}-x+12}{(x+4)(x-4)^2}=$$

$$=\frac{-4x+8}{(x+4)(x-4)^2}=\frac{4(2-x)}{(x+4)(x-4)^2}$$

3. $$\frac{(ax+1)^2-(x+a)^2}{(1-x^2)(1-a^2)}$$

Denominator is already in factorised form and if necessary later we can easily to continue factorising. We should concentrate on numerator as it is not in factorised form. The numerator is difference of two perfect squares and by factorising it we will look to obtain the same factor(s) as we have in denominator. That should be our main concept to simplify this expression.