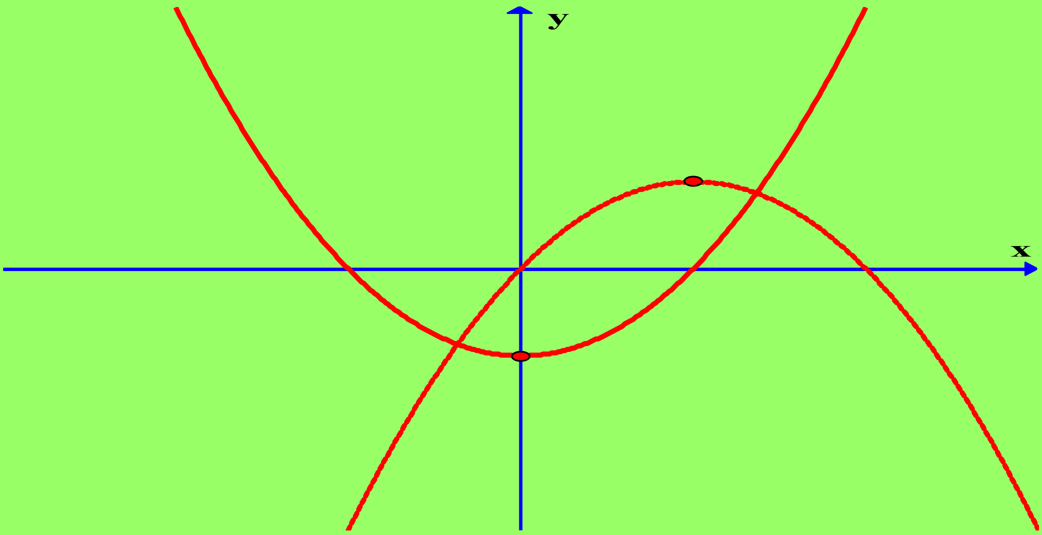
Concept based and Exam oriented

TUTORIAL LESSONS

Mathematics Year 10



In This Book

- Fully worked solutions to all questions
- Written explanation of theory and concepts for each question
- Over 1000 specifically designed study and exam style questions
- Strong focus on algebra from basic to the highest complexity
- Answers to most of your questions
- Help for homework and assessment
- Exam preparation through each lesson
- Systematic revision while study new topics

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About the book

Even though this book is fully compatible with the Victorian school Curriculum and contains all important topics and can also be very useful as supplementary study material in other states, the book is still different than any school text book as well as from most of the other mathematics books because all questions in this book are fully worked out and concisely explained in full details.

This book is designed to help you to find answers to, almost, all of your questions. Written by tutor with over 30 years tutoring experience, the book contains over 1000 fully worked questions with concise theoretical and conceptual explanation for each question. The intellectual base for this book is a systematic collection of questions asked by students over a long period of time. In order to provide students with all subject requirements the author created questions by adding other mathematical aspects such as: plenty algebra, application of mathematical theory and development of mathematical concepts and all this with strong tests and exam orientation. Finally, the wide range of different style of questions is helping students to learn mathematics with deeper understanding of theory and concepts that can be applied across the whole range of questions, rather than trying to apply some memorized patterns that usually can work on some certain questions. So this book will, definitely, help you:

- To find answers to most of your questions (even the most complex once)
- To do your homework and school tasks with easy and satisfactory way
- To be ready for your school assessments and final year exam
- To improve algebra skills
- To develop independent studying skills
- To get knowledge of the highest standard in the fastest time
- To understand and memorise even the most complex concepts in an easy way because most of the concepts are broken into simple steps and constantly revised throughout the book.
- To develop a high ability to apply theoretical knowledge and concepts in solving questions.
- To extend your educational ability and to increase motivation as you will discover that "knowledge is the power" and studying is not such hard work.

The book contains all important topics for Year 10 Mathematics, such as:

- Arithmetic and Algebra
- Linear Equations and Graphs
- Quadratic Expressions, Equations and Functions
- Index and Logarithm Laws. Exponential and Logarithmic Equations
- Pythagoras Theorem and Trigonometry

There are 41 tutorial lessons. Each lesson (session), contains in average about 25 fully worked out questions with concise explanation of theory and concept applied for each question. The firs (or first few) sessions of each topic were designed to present theory and basic concepts for that topic. The other sessions are practices sessions giving to students the best examples to get full understanding of the subject. Questions are not arranged according to their level of complexity, because all questions are fully worked out and concisely explained so that every student (regardless of current level of knowledge) can easily understand algebra, concept and theory applied on each questions. The other reason why questions are not graded is to motivate students to work across the entire book rather than to work selectively. Such arrangement and very detailed explanation of questions enable to students of all levels of knowledge to progress with this book to the highest possible standard.

Finally, this book is an attempt to save student's time and money seeing tutors around and to make available to everyone the best tutorial lessons that has been delivered to students over a few decades.

A short author’s message about how to study Mathematics

Choosing the right way of doing something is very solid base for success. As studying is a complex activity, so we have to be more conscious of choosing the right way of studying. This way is, generally, different from subject to subject and as Mathematics is one of very specific and quite complex subject, we have to take the right approach to that subject to ensure our success.

Definitely, we can’t study Mathematics by reading even not by memorising simple facts and formulas. Studying Mathematics requires permanent practice over long period. As we are progressing from grade to grade, we have to keep fresh our previous knowledge (theory, concepts, even facts and formulas) and to build up that knowledge from current program.

When we start studying a new chapter, of course, we have to “read” the book. During “reading” time we have to stop “reading” whenever is necessary to write down important words, facts and formulas or to draw a graph or some other appropriate drawing, even if that graph or drawing is already in the book. During that time we have to identify the meaning of new terminology, maybe some formulas and definitions, key facts, theoretical explanations, probably some new concepts and finally to identify previously learned knowledge we should apply in the current chapter. When you are happy with your understanding of what you “read” then you can move to the next phase of studying. That is practicing. But how someone can be sure that his/her understanding is good enough to move to practicing? Here I wouldn’t suggest any quiz or simple test questions trial. Instead, I would recommend a different technique. That is to visualise in front of you your friend, cousin or your parents and try to teach them of what you have been learning during “reading” time. If necessary, go back in book to refresh your memory or to check if you have missed something. The theory of each chapter of most of the books is usually written on just a few pages, so that “reading” process shouldn’t consume long time. One or two hours should qualify you to move to practicing phase.

Once when you start practicing, you can start from easier questions and go to the most complex once. For each question, before you start working, spend few minutes time to think about an appropriate concept and draw a simple strategy how to apply that concept. When you start work make sure that your work is inside your concept and strategy drawn at the beginning. If the concept, you have, doesn’t work for that question you should try to find out the clear reason why that concept is not good. That will help you to correct the concept and satisfactory to finish the question.

When due to move on another chapter, you should select some testing questions and to conduct short test of your understanding of current chapter. It is very useful, at the end of studying the chapter, once again, pretend to be a teacher and try to teach somebody. Compare your way of “teaching” somebody after “reading” phase and after practising phase of the end of studying the chapter and you will discover how big progress you made.

How to use this book

Thinking about method of how to study Mathematics, you already may have some ideas how to use this book. Even though you can use this book independently from any other books, still it would be better to study the chapter in your school and when you already have some knowledge of the chapter then to start using this book. Go over first session of the chapter to establish solid understanding of theory and important concepts. Even though you have all questions fully worked out, I am suggesting you to try to solve questions before checking working solution. After each question even if you solve the question correctly, go through worked solution with high attention as always there will be something there that you may learn. It is, also, recommended when going through worked solutions, to frequently reefer to theoretically session(s) to learn how the author is applying theory and concepts on practical questions.

Let start

Good luck

Contents

Index & Logarithm Laws. Exponential & Logarithmic Equations

S E S S I O N 8

LINEAR SIMULTANEOUS EQUATIONS

Concepts and Practice

b) Elimination Method

This method is giving us opportunity to get equation with one variable in simple form and on quicker way. To do that we have to make coefficients of the same variable equal in both equation and then by adding or subtracting equations to each other to eliminate that variable. To make coefficients equal, multiply one equation by coefficient of any variable from the other equation and multiply another equation by coefficient from previous equation. When we multiply equation by some number we have to multiply each term on both sides by that number. If both coefficients of variable that we like to eliminate are positive or both negative than we should subtract equations from each other. If one coefficient is positive and the other is negative than we have to ad equations to each other. Operation of adding or subtracting equations is the same like collecting like terms on both sides of equations.

Steps to apply elimination method:

1) Rearrange (if needed) both equation to form standard form of simultaneous equations.

$$\left.\begin{aligned} a_1x + b_1y = c_1 ----(1) \\ a_2x + b_2y = c_2 ----(2) \end{aligned}\right\} \quad \rightarrow \quad \text{Standard form}$$

2) Make decision which variable is easier to eliminate (usually that is variable with smaller coefficients).
3) Multiply equations simultaneously by coefficients of the variable you decided in step 2 to eliminate.
4) Subtract or ad equations to each other. Subtract if both coefficients positive or both negative. Ad equations to each other if one coefficient positive and another one is negative.
5) Solve the equation from step 4 to get value for one variable.
6) Substitute solution from step 5 in any of two equations and calculate value for another variable.

Example 1 (the same we used for substitution method)

$$\left.\begin{aligned} 3x + 2y = 6 ----(1) \\ 5x + 3y = 11 ----(2) \end{aligned}\right\}$$

Step 1 Equations are already in standard form and no need to rearrange them.

Step 2 It is better to eliminate y as this variable has smaller coefficients.

Step 3 We should multiply equation (1) by 3 and equation (2) by 2 to get the same coefficient of y

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SESSION 11

LINEAR EQUATIONS

Practice 3

Session 11 – Linear Equations – Practice 3 Worksheet

1. Solve the following linear equations and check your solutions by substituting solution in equation.

a) $\dfrac{x+2}{3}=3$

b) $\dfrac{2x+12}{7}=\dfrac{3x+5}{4}$

c) $\dfrac{1}{x}+b=c$

d) $\dfrac{1}{x}+\dfrac{1}{b}=\dfrac{1}{c}$

2. Solve the following simultaneous equations by substitution method and check your solutions by solving each equation by elimination method.

a) $\left.\begin{array}{l} x+2y=4 \\ 3x-y=5 \end{array}\right\}$

b) $\left.\begin{array}{l} 1-2x+4y=0 \\ 4y-3x=5 \end{array}\right\}$

c) $\left.\begin{array}{l} \dfrac{2x-y}{3}-x=4 \\ y=\dfrac{3}{2}x+2 \end{array}\right\}$

d) $\left.\begin{array}{l} ax+y=b \\ x-by=a \end{array}\right\}$

3. Solve the following simultaneous equations. Choose any appropriate method.

a) $\left.\begin{array}{l} 9x+4y=14 \\ 2x+y=2 \end{array}\right\}$

b) $\left.\begin{array}{l} 5x+3y=13 \\ 7x+2y=16 \end{array}\right\}$

c) $\left.\begin{array}{l} 7x=18+3y \\ 2x+5y=11 \end{array}\right\}$

d) $\left.\begin{array}{l} \dfrac{x}{5}+\dfrac{y}{2}=5 \\ x-y=4 \end{array}\right\}$

e) $\left.\begin{array}{l} \dfrac{y+2}{6}-\dfrac{y-4}{2}=\dfrac{x}{3} \\ \dfrac{4}{3}(y-1)-2x=-2 \end{array}\right\}$

f) $\left.\begin{array}{l} ax+by=p \\ bx-ay=q \end{array}\right\}$

g) $\left.\begin{array}{l} \dfrac{x}{a}+\dfrac{y}{b}=1 \\ \dfrac{x}{b}+\dfrac{y}{a}=1 \end{array}\right\}$

h) $\left.\begin{array}{l} \dfrac{2}{3}+\dfrac{x+1}{2x-y}=0 \\ \dfrac{x}{x+y}-\dfrac{y}{2x+2y}=4 \end{array}\right\}$

Session 11 – Linear Equations
Solutions Practice 3 Worksheet

1. a) b) c) d)

2. a) b) c) d)

3. a) b) c) d) e) f) g)

h)

$$\left.\begin{aligned}\frac{2}{3}+\frac{x+1}{2x-y}&=0\ldots\ldots\ldots\ldots\ldots(1)\\ \frac{x}{x+y}-\frac{y}{2x+2y}&=4\ldots\ldots\ldots(2)\end{aligned}\right\}\rightarrow\begin{cases}\text{Factorise denominator in}\,(2)\\ \text{to determine LCD}\end{cases}$$

$$\left.\begin{aligned}\frac{2}{3}+\frac{x+1}{2x-y}&=0\ldots\ldots\ldots\ldots\ldots(1)\quad /\times 3(2x-y)\\ \frac{x}{x+y}-\frac{y}{2(x+y)}&=4\ldots\ldots(2)\quad /\times 2(x+y)\end{aligned}\right\}\rightarrow \text{Get rid of fractions}$$

$$\left.\begin{aligned}2(2x-y)+3(x+1)&=0\ldots\ldots(1)\\ 2x-y&=4\times 2(x+y)\ldots\ldots\ldots(2)\end{aligned}\right\}-\rightarrow\begin{cases}\text{Expand and rearange}\\ \text{to standard form}\end{cases}$$

$$\left.\begin{aligned}4x-2y+3x+3&=0\ldots\ldots\ldots\ldots(1)\\ 2x-y&=8x+8y\ldots\ldots\ldots\ldots\ldots(2)\end{aligned}\right\}$$

$$\left.\begin{aligned}7x-2y&=-3\ldots\ldots\ldots\ldots(1)\\ -6x-9y&=0\ldots\ldots\ldots(2)\end{aligned}\right\}$$

$$(2)\rightarrow 9y=-6x$$

$$y=-\frac{2}{3}$$

$$y=-\frac{2}{3}\rightarrow(1)\Rightarrow 7x-2\times\left(-\frac{2}{3}\right)=-3$$

$$7x+\frac{4}{3}=-3$$

$$7x=-\frac{13}{3}$$

$$x=-\frac{13}{21}$$

S E S S I O N 15

APPLICATIO OF LINEAR RELATION

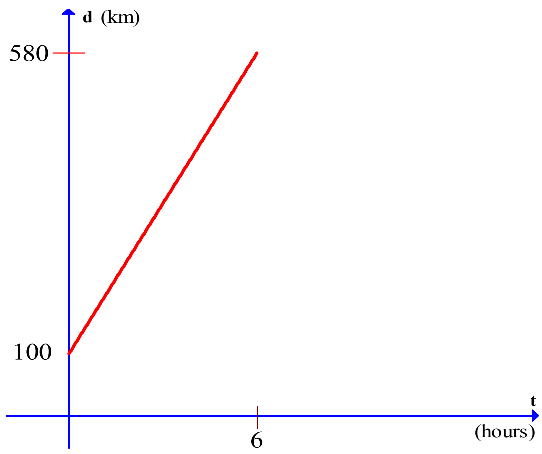
Practice 1

Session 15 – Application of Linear Relation – Practice 1

1. The length of rectangle is 10 cm longer than $\frac{2}{3}$ of its width. If the perimeter is 110 cm, find the dimensions of rectangle.

2. A tennis ticket costs $25 for children and $60 for adults. If 6000 people attended tennis matches for a particular day and the total takings were $300 010, determine the number of children and adults who attended tennis matches.

3. The graph below is distance-time graph representing a journey of a car from Melbourne to Sydney.



a) Find the gradient of the graph and intercept with vertical axes.
b) Write equation of the graph.
c) What constant speed was maintained throughout the journey?
d) If the journey had started 180 km from Melbourne and a constant speed of 95 km/h was maintained, Write new equation to represent this journey.
e) After 580 km until the end of journey, the constant speed was changed and the new journey is represented by the equation $5d - 490t - 2900 = 0$. What constant speed was maintained throughout this part of journey?

4. A company manufactures scientific calculators. Its fixed weekly cost $2025 and the cost to manufacture 1 calculator is $50.

a) Write a rule for the total weekly cost (C) to manufacture n calculators.
b) If company sales each calculator for $125, write a rule for its weekly revenue (R).
c) Sketch the graphs of C and R on the same set of axes.
d) Calculate the number of calculators n where the costs C are equal to the revenue R.

5. Two lines are given $y = 2x - 2$ and $y = -\frac{2x}{3} + 4$

a) Sketch graphs of both lines on the same set of axes. Label x and y intercepts with A and B for $y = 2x - 2$ and with C and D for $y = -\frac{2x}{3} + 4$. Label intersection point between those two lines by E.

b) Find the area of polygon $OBED$

c) In what ratio the x axis divides the area of triangle AED.

6. A train travelling between town A and town B for 10 hours. If speed of train is 80 km/h for half of journey, 60 km/h for one third and 40 km/h for the rest of trip, find the distance between A and B.

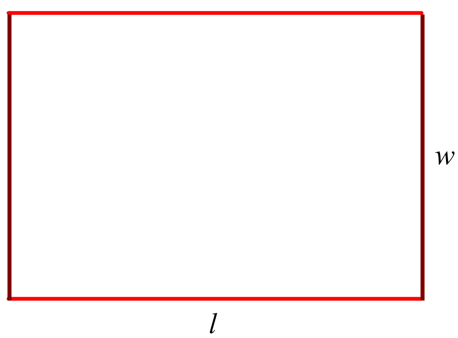
Session 15 – Application of Linear Relation
Solutions to Practice 1

How to think. Solving worded (analysis) questions requires application of mathematical knowledge to a particular problem that we have to solve. Solid understanding of mathematical concepts, theory, formulas and rules is one side of our success in this type of questions. The other side is clear understanding of problem we are solving. Connecting those two sides is actually application of mathematical knowledge. Assuming that we have solid understanding of mathematical concepts and theory, we will be then able to concentrate on text in our question. Depending of complexity of the question we may need to reed question several times until we get complete picture of the problem we are solving. When we reach full confidence of understanding the question, then we should identify particular mathematical knowledge that we need to bring to our question. The next step is to establish right concept of solving the question and to identify and distribute all necessary information (data) to a particular mathematical formulas and rules.

It is very helpful to translate text to some accurate drawing (whenever is possible) and to enter all necessary information in that drawing. This is, probably the most important step for analysis questions as our further work will be based on that drawing. Also it is important to keep working with notation as marked in drawing in order to stay in contest and to avoid some possible mistakes.

We are studying mathematics because we need mathematical knowledge for application across all subjects and disciplines for their better understanding. This is harder part of mathematics and need to be practice regularly

1. Dimensions of rectangle are length (l) and width (w). We will sketch this rectangle and mark as described in question.



It is expected to have simultaneous linear equations or to form one equation in l or w and to solve it. To form that equation we have to use information supplied in question and some knowledge of rectangle. So.
We know length in terms of with

$$l = \frac{2}{3}w + 10$$

We know perimeter of rectangle $P = 110$
We know formula for perimeter of rectangle
$P = 2l + 2w$
Now by substituting data from question in this formula we have:

$$110 = 2\left(\frac{2}{3}w + 10\right) + 2w$$

This is our equation for w and solving this equation we will get width of rectangle

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S E S S I O N   39

TRIGONOMETRY

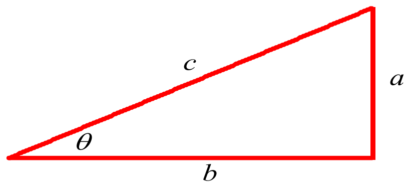
Unit Circle, Exact Values, Functions and Equations

Unit Circle, Exact Values, Functions and Equations

As we already know, trigonometry is a study of relations between angles and sides in right (any) angle triangle. For the triangle below the ratio $\frac{\textbf{\textit{opposite}}}{\textbf{\textit{hypotenuse}}}$ is $\sin\theta$; $\frac{\textbf{\textit{adjacent}}}{\textbf{\textit{hypotenuse}}}$ is $\cos\theta$ and $\frac{\textbf{\textit{opposite}}}{\textbf{\textit{adjacent}}}$ is $\tan\theta$. So we can write;

$\sin\theta = \frac{a}{c}$ $\cos\theta = \frac{b}{c}$ $\tan\theta = \frac{a}{b}$

See triangle below



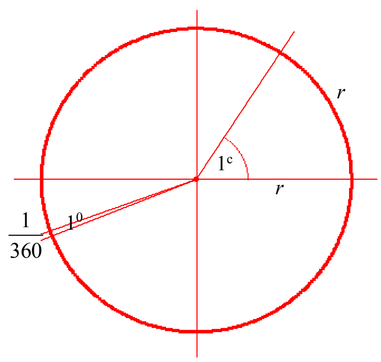
Note: $\sin\theta$ or $\cos\theta$ or $\tan\theta$ are just different names for different ratios. This is **not** $\sin\times\theta$, $\cos\times\theta$ or $\tan\times\theta$. When we write, for example, $\sin\theta = a$ than θ is an angle while a is the value of $\sin\theta$ (the ratio of opposite over hypotenuse).

From the above triangle and basic ratios we can derive two basic identities (formulas) that are very important and widely used in trigonometry.

$$\boxed{\sin^2\theta + \cos^2\theta = 1} \quad \text{and} \quad \boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$$

Units for angle

There are two different units to measure angles: ***Degree*** is an angle whose arc is $\frac{1}{360}$ of a perimeter of a circle. ***Radian*** is an angle whose arc is equal to radius of a circle. It is obvious that degree is much smaller angle and thoroughly $1^c = 57.096^0$ As there are two different units to measure angles than we have to know how to convert them to each other.



- To convert degrees to radians we have to multiply degrees by $\frac{\pi}{180}$
- To convert radians to degrees we have to multiply radians by $\frac{180}{\pi}$

Exampl 1 Convert 30^0 to radians

$$30\times\frac{\pi}{180}=\frac{\pi}{6}$$

Example 2 Convert 2^c to degrees

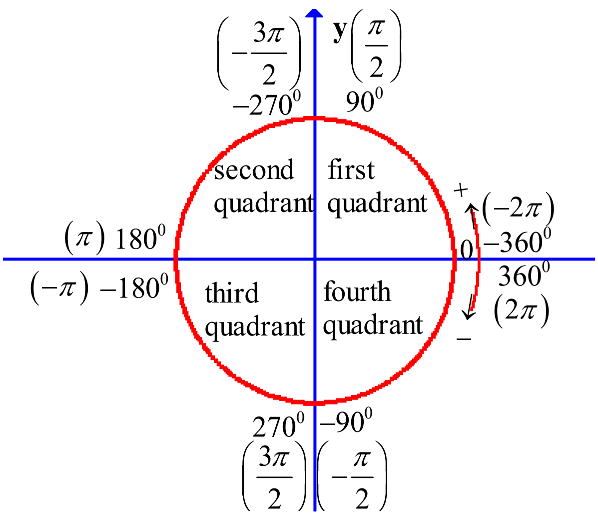
$$2^c\times\frac{180}{\pi}=114.59^0$$

Note: π is a constant number that has undefined number of decimal places without repetition or any patterns. The estimated value of π is 3.14…….. That is (π) ratio of area of a circle by area of a square of side of radius of that circle.

Unit circle

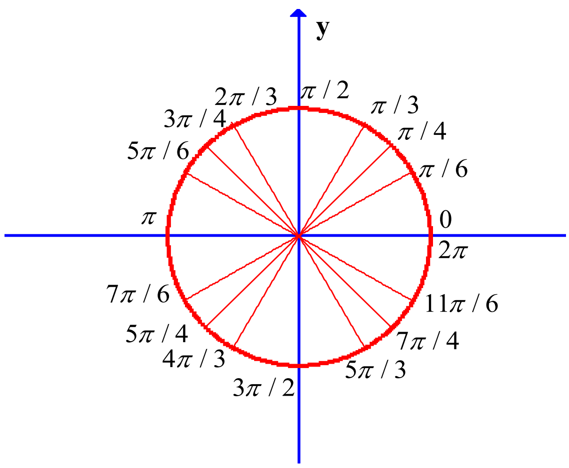
The circle of radius of one unit with the centre at origin is a unit circle. That is very powerful and practical tool in trigonometry and solid understanding and correct use of unit circle can help us to solve the most questions in trigonometry. Below are some features of unit circle.

Quadrants and positive and negative angles



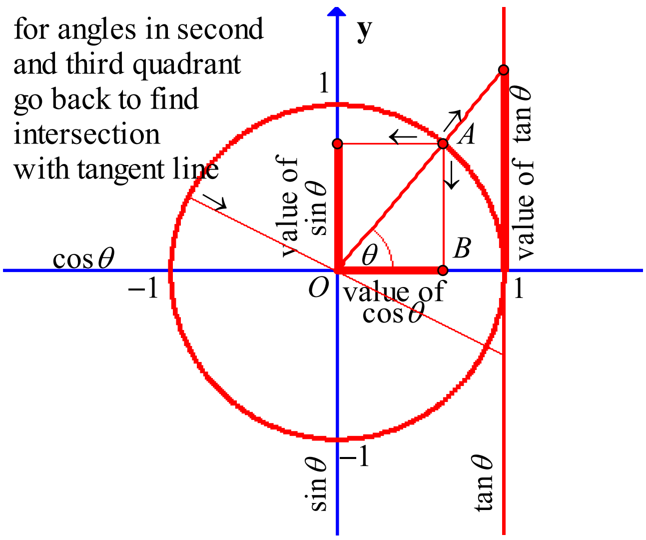
- All angles measured in anti clock wise direction are positive
- Angles measured in clock wise direction are negative

Some angles in first quadrant and other angles by symmetry



Angles in first quadrant are the most common angles in trigonometry as well the angles in other quadrants constructed by symmetry from first quadrant. We have to understand correct position of those angles.

Unit circle and values of $\sin\theta$***,*** $\cos\theta$ ***and*** $\tan\theta$



Values of $\sin\theta$ for any angle are on vertical axis between points – 1 and 1

Values of $\cos\theta$ for any angle are on horizontal axis between – 1 and 1

Values of $\tan\theta$ for any angle are on tangent line to the circle through point 1 and they can take value from $-\infty$ to $+\infty$.

So we can write:

$$-1 \le \begin{Bmatrix}\sin\theta \\ \cos\theta\end{Bmatrix} \le 1;\ -\infty \le \tan\theta \le +\infty$$

Proof: Consider triangle $\triangle \boldsymbol{OBA}$

Then $\cos\theta = \dfrac{\overline{\boldsymbol{OB}}}{\overline{\boldsymbol{OA}}}$

Since $\overline{\boldsymbol{OA}} = 1$ (radius of unit circle then $\cos\theta = \overline{\boldsymbol{OB}}$

Exact Values

Generally we should use calculator to find value of $\sin\theta$, $\cos\theta$ or $\tan\theta$ for any angle θ. Usually, on this way we are getting estimated (decimal) value for the most of angles. However, as some angles are very common in use, we need to work with exact values.

Those angles are: 0^0, 30^0 or $\left(\frac{\pi}{6}\right)$, 45^0 or $\left(\frac{\pi}{4}\right)$, 60^0 or $\left(\frac{\pi}{3}\right)$, 90^0 or $\left(\frac{\pi}{2}\right)$. The exact values for those angles are given in the table below and we have to memorise them.

Angle / Ratio	0^0	30^0 or $\left(\frac{\pi}{6}\right)$	45^0 or $\left(\frac{\pi}{4}\right)$	60^0 or $\left(\frac{\pi}{3}\right)$	90^0 or $\left(\frac{\pi}{2}\right)$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

The angles in this table are angles in first quadrant. By symmetry of unit circle, for each of those angles we can determine corresponding angles in other quadrants and consequently exact values for each ratio. It is obvious that exact values will be the same numbers from table above but some of them will be negative depending in which quadrant the angle is.

The table of exact values for angles in all four quadrants is given below. The table, also, contains conversion between degrees and radians for those angles.

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S E S S I O N 41

Trigonometry

Practice 2

Session 41 – Trigonometry worksheet 2

1 For the triangle shown bellow

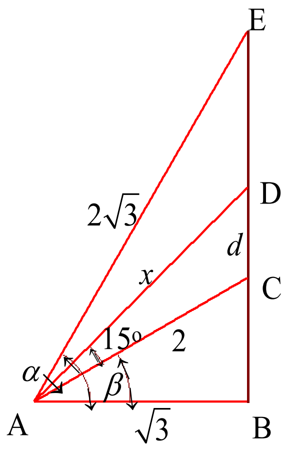
a) Find exact value for $\cos\alpha$ and hence find angle α. Note: angle α is $\measuredangle EAB$

b) Find exact value for $\cos\beta$ (angle β is $\measuredangle CAB$). Hence find angle β.

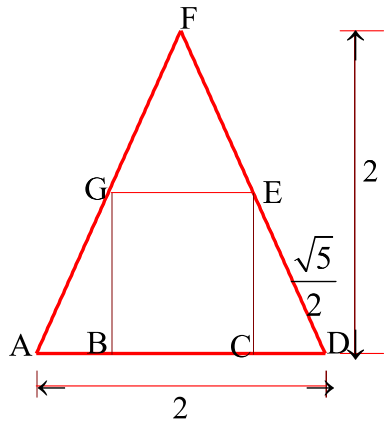
c) Find exact value of x.

d) Find exact value of d.

e) Find exact length of DE



2. Find the area of the rectangle inscribed inside an isosceles triangle as shown on the diagram. The length of base and the height of triangle are 2 units each. The length of DE is $\frac{\sqrt{5}}{2}$ units (use trigonometry as much as possible).



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Session 41 – Solution to trigonometry worksheet 2

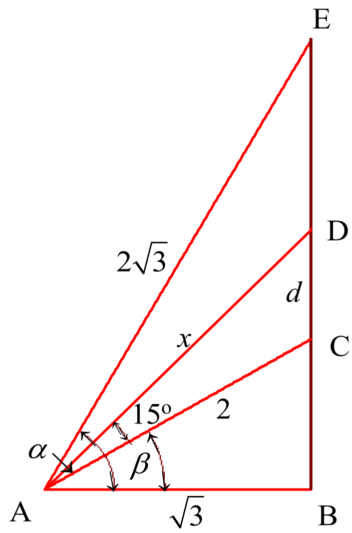
1.

a) Using right angled triangle ABE we have

$$\cos\alpha = \frac{\sqrt{3}}{2\sqrt{3}} = \frac{1}{2}$$

We can use calculator to get value for angle α , but as an exact value required we should use table of exact value

$$\alpha = \cos^{-1}\left(\frac{1}{2}\right) = 60^o$$



b) Using right angled triangle ABC we have

$$\cos\beta = \frac{\sqrt{3}}{2}$$

$$\beta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30^o$$

c) x is hypotenuse of triangle ABD. We know one shorter side of that triangle and using result from part (b) we can calculate the angle at A.

$$\cos\left(\beta + 15^o\right) = \frac{\sqrt{3}}{x}$$

$$\beta = 30^o \qquad \text{from part (b)}$$

$$\cos 45^o = \frac{\sqrt{3}}{x}$$

$$x\cos 45^o = \sqrt{3}$$

$$x = \frac{\sqrt{3}}{\cos 45^o} = \frac{\sqrt{3}}{\frac{1}{\sqrt{2}}} = \sqrt{2}\sqrt{3} = \sqrt{6}$$

d) We can't find value of d directly from any triangle because d is not side of any right angled triangle (d is a side of ACD triangle which is not right angled triangle). On the other hand d is part of BD and BD is shorter side of right angled triangle ABD. This is still not enough to get d as we don't know the length of BC. Since BC is a shorter side of right angled triangle ABC then we can calculate length of BC from that triangle.

This part of question requires involvement of two right angled triangles and that is very common style of questions in application of trigonometry for this course.

We will start with expressing length of d (see diagram).

$$d = \overline{BD} - \overline{BC}$$

$$\tan 45^o = \frac{\overline{BD}}{\sqrt{3}} \qquad \text{From } \Delta ABD$$

$$\overline{BD} = \sqrt{3}\tan 45^o = \sqrt{3}\times 1 = \sqrt{3}$$

$$\tan 30^o = \frac{\overline{BC}}{\sqrt{3}} \qquad \text{From } \Delta ABC$$

$$\overline{BC} = \sqrt{3}\tan 30^o = \sqrt{3}\times\frac{1}{\sqrt{3}} = 1$$

$$d = \sqrt{3} - 1$$

e) This part of question is similar to part (d). So

$$\overline{DE} = \overline{BE} - \overline{BD}$$

$$\tan 60^o = \frac{\overline{BE}}{\sqrt{3}} \qquad \text{From } \Delta ABE$$

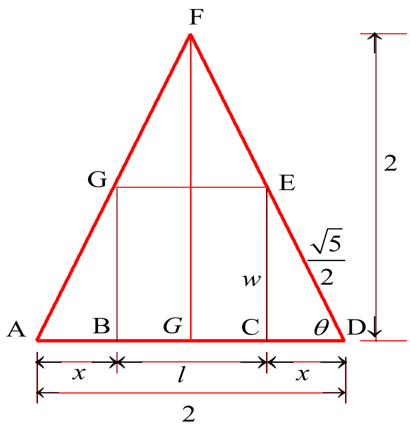
$$\overline{BE} = \sqrt{3}\tan 60^o = \sqrt{3}\times\sqrt{3} = 3$$

$$\tan 45^o = \frac{\overline{BD}}{\sqrt{3}} \qquad \text{From } \Delta ABD$$

$$\overline{BD} = \sqrt{3}\tan 45^o = \sqrt{3}\times 1 = \sqrt{3}$$

$$\overline{DE} = 3 - \sqrt{3}$$

2.



Area of required rectangle is $A = wl$. We have to find values for w and l.

Let $\overline{AB} = \overline{CD} = x$ and $\sphericalangle GDF = \sphericalangle CDE = \theta$

We can express l in terms of x since we know length of base of triangle $(l = 2 - 2x)$. The idea is to express w, also, in terms of x and on that way to get area in terms of x only. When we have w in terms of x, then we can calculate x from triangle CDE because we know hypotenuse $\left(\frac{\sqrt{5}}{2}\right)$ of that triangle. The major problem in this question is to see connection between w and x. For that purpose, we have to consider two similar triangles; ΔGDF and ΔCDE and to write trigonometric ratio for $\tan\theta$ for each of them.

$A = wl$

$\tan\theta = \dfrac{\overline{GF}}{\overline{GD}} = \dfrac{2}{1} = 2$ From ΔGDF

$\tan\theta = \dfrac{\overline{CE}}{\overline{CD}} = \dfrac{w}{x}$ From ΔCDE

$\dfrac{w}{x} = 2$ Equalizing right hand sides.

$w = 2x$

$A = 2x(2-2x)$ Substituting $w = 2x$ and $l = 2-2x$ in $A = wl$

$(2x)^2 + x^2 = \left(\dfrac{\sqrt{5}}{2}\right)^2$ Pythagoras theorem for ΔCDE to find x.

$4x^2 + x^2 = \dfrac{5}{4}$

$5x^2 = \dfrac{5}{4}$

$x^2 = \dfrac{1}{4}$

$x = \sqrt{\dfrac{1}{4}}$

$x = \dfrac{1}{2}$

$A = 2\times\dfrac{1}{2}\left(2-2\times\dfrac{1}{2}\right) = 2-1$

$A = 1$ Square units

Note: Question, also, can be solved by applying ratios for two similar triangles. In that case we need to calculate hypotenuse $\left(\overline{DF}\right)$ of ΔGDF by setting up Pythagoras theorem for that triangle. So:

$\dfrac{\overline{DF}}{\overline{DE}} = \dfrac{\overline{GF}}{\overline{CE}}$ and $\dfrac{\overline{DF}}{\overline{DE}} = \dfrac{\overline{GD}}{\overline{CD}}$

$\dfrac{\sqrt{5}}{\frac{\sqrt{5}}{2}} = \dfrac{2}{w}$ $\dfrac{\sqrt{5}}{\frac{\sqrt{5}}{2}} = \dfrac{1}{x}$

$\dfrac{2\sqrt{5}}{\sqrt{5}} = \dfrac{2}{w}$ $\dfrac{2\sqrt{5}}{\sqrt{5}} = \dfrac{1}{x}$

$1 = \dfrac{1}{w}$ $2 = \dfrac{1}{x}$

$w = 1$ $x = \dfrac{1}{2}$

$l = 2-2\times\dfrac{1}{2} = 2-1 = 1$

$A = 1\times1 = 1$ Square units.